icpc international collegiate programming contest

ICPC North America Regionals 2019

ICPC North Central NA Regional Contest

November 9, 2019

Official Problem Set

Michigan Tech | site host

icpc international collegiate programming contest

icpc.foundation

JET BRAINS | icpc global programming tools sponsor

TWO SIGMA | icpc north america sponsor

Problems

This page is intentionally left blank.

Problem A
Weird Flecks, But OK

An artist who wanted to create an installation where his works appeared to be floating in midair has cast a large cube of clear acrylic to serve as a base. Unfortunately, during the casting, some small flecks of dirt got into the mix, and now appear as a cluster of pinpoint flaws in the otherwise clear cube.

He wants to drill out the portion of the cube containing the flaws so that he can plug the removed volume with new, clear acrylic. He would prefer to do this in one drilling step. For stability's sake, the drill must enter the cube perpendicular to one of its faces.

Given the (x, y, z) positions of the flaws, and treating the size of the flaws as negligible, what is the smallest diameter drill bit that can be used to remove the flaws in one operation??

The drill may enter any one of the cube faces, but must be positioned orthogonally to the face.

Input

The first line of input will contain an integer N denoting the number of flaws. $3 \le N \le 5\,000$

This is followed by N lines of input, each containing three real numbers in the range $-1\,000.0 \ldots 1\,000.0$, denoting the (x, y, z) coordinates of a single flaw. Each number will contain at most 6 digits following a decimal point. The decimal point may be omitted if all succeeding digits are zero.

Output

Print the diameter of the smallest drill bit that would remove all the flaws.

The answer is considered correct if the absolute or relative error is less than 10^{-4}

Sample Input 1	Sample Output 1
3 1.0 0.0 1.4 -1.0 0.0 -1.4 0.0 1.0 -0.2	2.0000000000

Sample Input 2	Sample Output 2
5 1.4 1.0 0.0 -0.4 -1.0 0.0 -0.1 -0.25 -0.5 -1.2 0.0 0.9 0.2 0.5 0.5	2.0000000000

Sample Input 3

```
8
435.249 -494.71 -539.356
455.823 -507.454 -539.257
423.394 -520.682 -538.858
446.507 -501.953 -539.37
434.266 -503.664 -560.631
445.059 -549.71 -537.501
449.65 -506.637 -513.778
456.05 -499.715 -561.329
```

Sample Output 3

```
49.9998293198
```

Problem B
Code Names

You are given W, a set of N words that are anagrams of each other. There are no duplicate letters in any word. A set of words $S \subseteq W$ is called "swap-free" if there is no way to turn a word $x \in S$ into another word $y \in S$ by swapping only a single pair of (not necessarily adjacent) letters in x. Find the size of the largest swap-free set S chosen from the given set W.

Input

The first line of input contains an integer N ($1 \le N \le 500$). Following that are N lines each with a single word. Every word contains only lowercase English letters and no duplicate letters. All N words are unique, have at least one letter, and every word is an anagram of every other word.

Output

Output the size of the largest swap-free set.

Sample Input 1	Sample Output 1
6 abc acb cab cba bac bca	3

Sample Input 2	Sample Output 2
11 alerts alters artels estral laster ratels salter slater staler stelar talers	8

Sample Input 3	Sample Output 3
6 ates east eats etas sate teas	4

Problem C
New Maths

A Cam from a Babbage Analytical Engine

"Drat!" cursed Charles. "This stupid carry bar is not working in my Engine! I just tried to calculate the square of a number, but it's wrong; all of the carries are lost."

"Hmm," mused Ada, "arithmetic without carries! I wonder if I can figure out what your original input was, based on the result I see on the Engine."

Carryless addition, denoted by $\oplus$, is the same as normal addition, except any carries are ignored (in base 10). Thus, $37 \oplus 48$ is 75, not 85.

Carryless multiplication, denoted by $\otimes$, is performed using the schoolboy algorithm for multiplication, column by column, but the intermediate additions are calculated using *carryless addition*. More formally, Let $a_m a_{m-1} \ldots a_1 a_0$ be the digits of a, where a_0 is its least significant digit. Similarly define $b_n b_{n-1} \ldots b_1 b_0$ be the digits of b. The digits of $c = a \otimes b$ are given by the following equation:

$$c_k = (a_0 b_k \oplus a_1 b_{k-1} \oplus \cdots \oplus a_{k-1} b_1 \oplus a_k b_0) \bmod 10,$$

where any a_i or b_j is considered zero if $i > m$ or $j > n$. For example, $9 \otimes 1\,234$ is $9\,876$, $90 \otimes 1\,234$ is $98\,760$, and $99 \otimes 1\,234$ is $97\,536$.

Given N, find the smallest positive integer a such that $a \otimes a = N$.

Input

The input consists of a single line with a positive integer N, with at most 25 digits and no leading zeros.

Output

Print, on a single line, the least positive number a such that $a \otimes a = N$. If there is no such a, print '-1' instead.

Examples

Sample Input 1	Sample Output 1
6	4

Sample Input 2	Sample Output 2
149	17

Sample Input 3	Sample Output 3
123476544	11112

Sample Input 4	Sample Output 4
15	-1

This page is intentionally left blank.

Problem D
Some Sum

Your friend has secretly picked N consecutive positive integers between 1 and 100, and wants you to guess if their sum is even or odd.

If the sum must be even, output '`Even`'. If the sum must be odd, output '`Odd`'. If the sum could be even or could be odd, output '`Either`'.

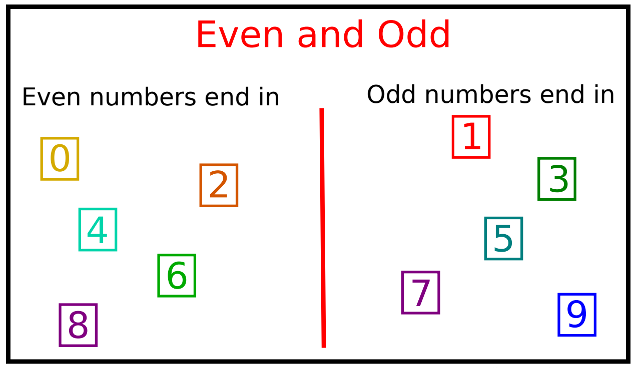


Illustration by Larry Pyeatt

Input

The input is a single integer N with $1 \leq N \leq 10$.

Output

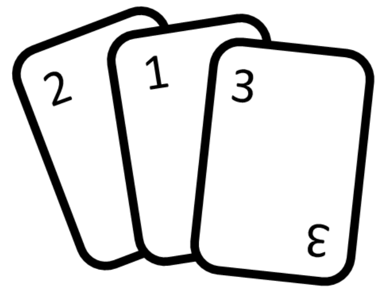
Output a single word. The word should be '`Even`', '`Odd`', or '`Either`', according to the rules given earlier.

Sample Input 1	Sample Output 1
`1`	`Either`

Sample Input 2	Sample Output 2
`2`	`Odd`

This page is intentionally left blank.

Problem E
Early Orders



You are given a list of integers $x_1, x_2, \ldots, x_n$ and a number k. It is guaranteed that each i from 1 to k appears in the list at least once.

Find the lexicographically smallest subsequence of x that contains each integer from 1 to k exactly once.

Input

The first line will contain two integers n and k, with $1 \le k \le n \le 200\,000$. The following n lines will each contain an integer x_i with $1 \le x_i \le k$.

Output

Write out on one line, separated by spaces, the lexicographically smallest subsequence of x that has each integer from 1 to k exactly once.

Examples

Sample Input 1	Sample Output 1
6 3 3 2 1 3 1 3	2 1 3

Sample Input 2	Sample Output 2
10 5 5 4 3 2 1 4 1 1 5 5	3 2 1 4 5

This page is intentionally left blank.

Problem F
Pulling Their Weight

To save money, Santa Claus has started hiring other animals besides reindeer to pull his sleigh via short term ‘gig’ contracts. As a result, the actual animals that show up to pull his sleigh for any given trip can vary greatly in size.

Last week he had 2 buffalo, 37 voles and a schnauzer. Unfortunately, both buffalo were hitched on the left side and the entire sleigh flipped over in mid-flight due to the weight imbalance.

To prevent such accidents in the future, Santa needs to divide the animals for a given trip into two groups such that the sum of the weights of all animals in one group equals the sum of the weights of all animals in the other. To make the hitching process efficient, Santa is seeking an integer target weight t such that all animals that are lighter than t go in one group and those heavier than t go into the other. If there are multiple such t, he wants the smallest one. There’s one small wrinkle: what should be done if there some animals have weight exactly equal to t? Santa solves the problem this way: if there are an even number of such animals, he divides them equally among the two groups (thus distributing the weight evenly). But if there are an odd number of such animals, then one of those animals is sent to work with the elves to make toys (it is not put in either group), and the remaining (now an even number) are divided evenly among the two groups.

Buffalo, vole, and schnauzer. Original images from Wikipedia.

Input

Input describes a list of animals’ weights. The first line contains an integer m ($2 \leq m \leq 10^5$), indicating the number of animals. The next m lines each have a positive integer. These are the weights of the animals (in ounces). Animals weighing more than 20 000 ounces are too big to pull the sleigh so no given weight will exceed this maximum.

Output

Output the smallest integer target weight t, as described above. It’s guaranteed that it is possible to find such an integer.

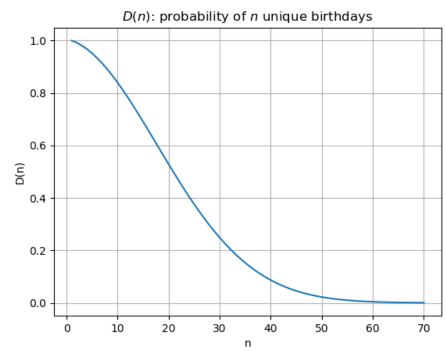
Sample Input 1	Sample Output 1
4 3 6 1 2	4

Sample Input 2	Sample Output 2
4 11 8 3 10	10

Sample Input 3	Sample Output 3
2 99 99	99

Problem G
Birthday Paradox

The Birthday Paradox is the name given to the surprising fact that if there are just 23 people in a group, there is a greater than 50% chance that a pair of them share the same birthday. The underlying assumptions for this are that all birthdays are equally likely (which isn't quite true), the year has exactly 365 days (which also isn't true), and the people in the group are uniformly randomly selected (which is a somewhat strange premise). For this problem, we'll accept these assumptions.



The probability of n unique birthdays among n people.

Consider what we might observe if we randomly select groups of $P = 10$ people. Once we have chosen a group, we break them up into subgroups based on shared birthdays. Among *many* other possibilities, we might observe the following distributions of shared birthdays:

- all 10 have different birthdays, or
- all 10 have the same birthday, or
- 3 people have the same birthday, 2 other people have the same birthday (on a different day), and the remaining 5 all have different birthdays.

Of course, these distributions have different probabilities of occurring.

Your job is to calculate this probability for a given distribution of people sharing birthdays. That is, if there are P people in a group, how probable is the given distribution of shared birthdays (among all possible distributions for P people chosen uniformly at random)?

Input

The first line gives a number n where $1 \leq n \leq 365$. The second line contain integers c_1 through c_n, where $1 \leq c_i \leq 100$ for all c_i. The value c_i represents the number of people who share a certain birthday (and whose birthday is distinct from the birthdays of everyone else in the group).

Output

Compute the probability b of observing a group of people with the given distribution of shared birthdays. Since b may be quite small, output instead $\log_{10}(b)$. Your submission's answer is considered correct if it has an absolute or relative error of at most 10^{-6} from the judge's answer.

Explanations

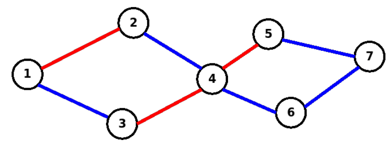
The first sample case shows $P = 2$ people with distinct birthdays. The probability of this occurring is $b = 364/365 \approx 0.9972602740$, and $\log_{10}(b) \approx -0.001191480807419$.

The second sample case represents the third example in the list given earlier with $P = 10$ people. In this case, the probability is $b \approx 0.0000489086$, and $\log_{10}(b) \approx -4.310614508857128$.

Sample Input 1	Sample Output 1
2 1 1	-0.001191480807419

Sample Input 2	Sample Output 2
7 1 1 2 1 3 1 1	-4.310614508857128

Problem H
On Average They're Purple



Alice and Bob are playing a game on a simple connected graph with N nodes and M edges.

Alice colors each edge in the graph red or blue.

A path is a sequence of edges where each pair of consecutive edges have a node in common. If the first edge in the pair is of a different color than the second edge, then that is a "color change."

After Alice colors the graph, Bob chooses a path that begins at node 1 and ends at node N. He can choose any path on the graph, but he wants to minimize the number of color changes in the path. Alice wants to choose an edge coloring to maximize the number of color changes Bob must make. What is the maximum number of color changes she can force Bob to make, regardless of which path he chooses?

Input

The first line contains two integer values N and M with $2 \le N \le 100\,000$ and $1 \le M \le 100\,000$. The next M lines contain two integers a_i and b_i indicating an undirected edge between nodes a_i and b_i ($1 \le a_i, b_i \le N$, $a_i \ne b_i$).

All edges in the graph are unique.

Output

Output the maximum number of color changes Alice can force Bob to make on his route from node 1 to node N.

Sample Input 1	Sample Output 1
3 3 1 3 1 2 2 3	0

Sample Input 2	Sample Output 2
7 8 1 2 1 3 2 4 3 4 4 5 4 6 5 7 6 7	3

This page is intentionally left blank.

Problem I
Full Depth Morning Show

All boring tree-shaped lands are alike, while all exciting tree-shaped lands are exciting in their own special ways. What makes Treeland more exciting than the other tree-shaped lands are the raddest radio hosts in the local area: Root and Leaf. Every morning on FM 32.33 (repeating of course), Root and Leaf of The Full Depth Morning Show serve up the hottest celebrity gossip and traffic updates.

The region of Treeland is made of n cities, connected by $n - 1$ roads such that between every pair of cities there is exactly one simple path. The ith road connects cities u_i and v_i, and has a toll of w_i.

To reward their loyal listeners, The Full Depth Morning Show is giving away a number of travel packages! Root and Leaf will choose $n - 1$ lucky residents from the city that sends them the most fan mail. Each of those residents then gets a distinct ticket to a different city in Treeland.

Each city in Treeland has its own tax on prizes: t_i. Let $d_{u,v}$ be the sum of the tolls on each road on the only simple path from city u to v. For a trip from city u to city v, the cost of that trip is then $(t_u + t_v)d_{u,v}$.

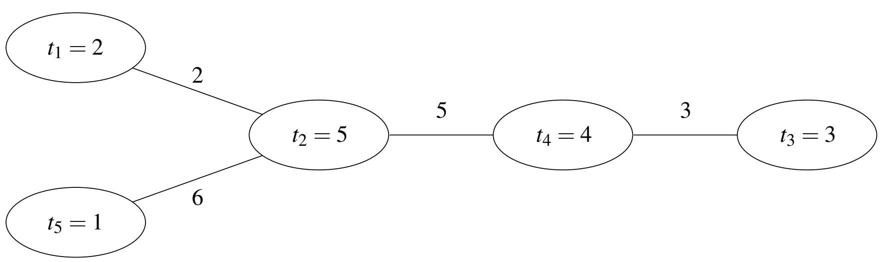


Figure I.1: The map of Treeland corresponding to the first sample input.

The shock jocks haven't quite thought through how much their prize is worth. They need to prepare a report to the radio executives, to summarize the expected costs. For each city that could win the prize, what is the total cost of purchasing all the tickets?

Input

The first line of input is a single integer n ($1 \le n \le 100\,000$). The next line has n space-separated integers t_i ($1 \le t_i \le 1\,000$), the tax in each city. The following $n - 1$ lines each have 3 integers, u_i, v_i, w_i, meaning the ith road connects cities u_i and v_i ($1 \le u_i, v_i \le n$), with a toll of w_i ($1 \le w_i \le 1\,000$).

Output

Output n lines. On the ith line, output a single integer: the cost of purchasing tickets if city i wins the contest.

Sample Input 1

```
5
2 5 3 4 1
1 2 2
2 4 5
4 3 3
5 2 6
```

Sample Output 1

```
130
159
191
163
171
```

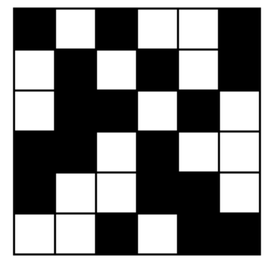
Sample Input 2

```
6
4 3 3 4 3 3
1 3 2
2 1 1
1 4 6
4 5 6
6 4 2
```

Sample Output 2

```
209
206
232
209
336
232
```

Problem J
This Ain't Your Grandpa's Checkerboard



You are given an n-by-n grid where each square is colored either black or white. A grid is *correct* if all of the following conditions are satisfied:

- Every row has the same number of black squares as it has white squares.
- Every column has the same number of black squares as it has white squares.
- No row or column has 3 or more consecutive squares of the same color.

Given a grid, determine whether it is *correct*.

Input

The first line contains an integer n ($2 \le n \le 24$; n is even). Each of the next n lines contains a string of length n consisting solely of the characters 'B' and 'W', representing the colors of the grid squares.

Output

If the grid is *correct*, print the number 1 on a single line. Otherwise, print the number 0 on a single line.

Sample Input 1

```
4
WBBW
WBWB
BWWB
BWBW
```

Sample Output 1

```
1
```

Sample Input 2

```
4
BWWB
BWBB
WBBW
WBWW
```

Sample Output 2

```
0
```

Sample Input 3

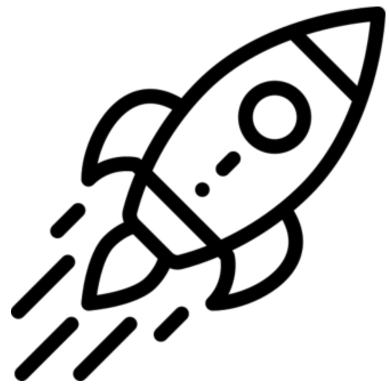
```
6
BWBWWB
WBWBWB
WBBWBW
BBWBWW
BWWBBW
WWBWBB
```

Sample Output 3

```
0
```

Sample Input 4	Sample Output 4
6 WWBBWB BBWWBW WBWBWB BWBWBW BWBBWW WBWWBB	1

Problem K
Solar Energy



You are planning to travel in interstellar space in the hope of finding habitable planets. You have already identified N stars that can recharge your spaceship via its solar panels. The only work left is to decide the orientation of the spaceship that maximizes the distance it can travel.

Space is modeled as a 2D plane, with the Earth at the origin. The spaceship can be launched from the Earth in a straight line, in any direction. Star i can provide enough energy to travel T_i distance if the spaceship is launched at an angle of a_i with the x-axis. If the angle is not perfectly aligned, then the spaceship gets less energy. Specifically, if the launch direction makes an angle of a with the x-axis, then it gets enough energy to travel distance of

$$\max(0, T_i - s_i \cdot \mathrm{dist}(a_i, a))$$

from star i, where $\mathrm{dist}(a, b)$ is the minimum radians needed to go from angle a to b. The distance that the spaceship can travel is simply the sum of the distances that each star contributes. Find the maximum distance T that the starship can travel.

Input

The first line contains the value N, $1 \leq N \leq 10^5$. Following this are N lines each containing three real numbers T_i, s_i, and a_i, with $0 < T_i \leq 1\,000$, $0 \leq s_i \leq 100$, and $0 \leq a_i < 2\pi$. All real numbers in the input have at most 6 digits after the decimal point.

Output

On a single line output the maximum distance the spacecraft can travel. Your answer is considered correct if it has an absolute or relative error of at most 10^{-6}.

Examples

Sample Input 1

```
2
100 1 1
100 1 1.5
```

Sample Output 1

```
199.500000
```

Sample Input 2

```
4
100 1 0.5
200 1 1
100 0.5 1.5
10 2 3
```

Sample Output 2

```
405.500000
```

This page is intentionally left blank.