

# NCNA 2020-21 Solution Slides

NCNA Judges

## Problem Set Developers

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- Pasha Kazatsker

## Problem

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$$V = \sqrt{(R * (S + .16)) / .067.}$$

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$$V = \sqrt{(R * (S + .16)) / .067.}$$

## Solution

Straightforward.

## Pitfalls

- Not rounding properly.
- Not reading to end-of-input properly.

## Java Code

```
public static void main(String[] args) {
    Scanner scan = new Scanner(System.in);
    while (scan.hasNext()) {
        double R = scan.nextDouble();
        double S = scan.nextDouble();
        double V = Math.sqrt(R * (S + 0.16) / 0.067);
        long ans = Math.round(V);
        System.out.println(ans);
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Statistics: 235 submissions, 77 accepted.



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- Finding the “most recent preceding value for  $s$  outside of range  $[i + t_f, i + t_r]$ ” is prone to bugs. You are better off iterating backwards to ensure you do it correctly. This is worst-case  $O(n^2)$ , though with the input bounds this should be fine. I also didn't create any cases to force  $O(n^2)$  runtime on look-back solutions (oops).



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- Confusion on inclusivity of “falls between” (it doesn't matter since  $t_f$  and  $t_r$  are of the form  $0.x5$ , where  $x \in \{0, \dots, 9\}$ , and speed is given to the first decimal place.).

## Pitfalls, cont.

- It is possible  $t_f > 0.5$  or  $t_r < 0.5$ , so avoid library rounding.

## Pitfalls, cont.

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Statistics: 339 submissions, 40 accepted.

## D - Substring Characters

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For each string in input, determine the number of unique proper contiguous substrings that have the same set of characters as the input string and contain no proper substrings also containing the same set of characters as the input string.

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One straightforward solution:

- 1 Given a starting index  $i$ , find the smallest  $j$  such that the substring from  $i$  to  $j$  has all unique characters; call it  $\text{ending}(i)$ .

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- 2 Add into a set/hashset all proper substrings  $[i, j]$  where  $\text{ending}(i + 1) \neq \text{ending}(i)$ .
- 3 Report the number of strings in the set.

## Pitfalls

- I/O. Again. Many submissions did not pass sample data.



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Statistics: 105 submissions, 36 accepted.

## Problem

Given a set of “facts”, resembling function calls, and a set of queries, determine how many facts are matched by each query, following the stated rules for matching.

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Given a query, check each fact to determine if it matches.

## Pitfalls

Inproper handling of spaces between tokens. Testcase `handmade2.in`:

```
fact(a,b,c)
```

```
fact ( a , b,c)
```

```
fact(,_,_) // should be 2
```

## Pitfalls

Inproper logic. Testcase handmade1.in:

```
test(arg1, arg1)
```

```
test(arg1, arg2)
```

```
test(__arg2, __arg2) // 1
```

```
test(__arg1, __arg2) // 2
```

```
test2(_, __) // 0
```

```
test(_, __) // 2
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Statistics: 96 submissions, 24 accepted.

## Problem

Given specifications for a set of points on a 2D plane representing shoelace holes, determine the number of valid symmetric shoelace patterns that result in shoelace lengths between a lower bound  $f_{min}$  and upper bound  $f_{max}$ .

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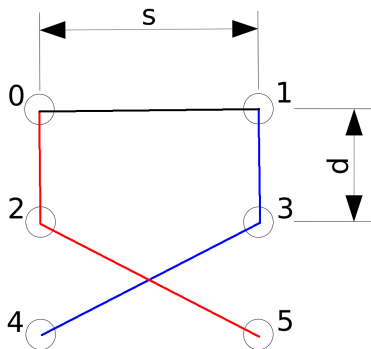
## Solution

- The first task is to get an upper bound on the number of possible valid lacing patterns.



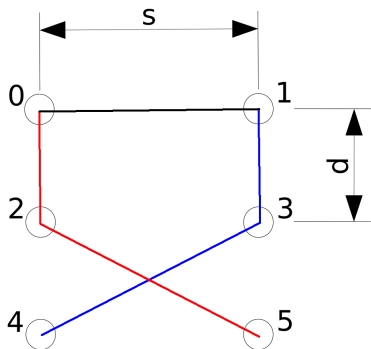
## J - Ada Loveslaces

Since hole 0 must go to 1 or 1 to 0, and we must start and end at  $2N - 2$  and  $2N - 1$ , we can figure out the pattern from  $2N - 2$  to 0 or 1, then the rest of the holes are a reflection.



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We can only choose one of  $\{2, 3\}$ ,  $\{4, 5\}$ ,  $\dots$ ,  $\{2N - 4, 2N - 3\}$  on the way from  $2N - 2$  to 0 or 1 so that we may complete the reflection.

## Solution

A loose upper bound can be determined as follows. Consider the pattern between  $2N - 2$  and whichever of 0 or 1 the lace goes through first.

- Of each pair  $\{0, 1\}, \{2, 3\}, \dots, \{2N - 4, 2N - 3\}$  the lace goes through either the left hole, the right hole, or neither hole (for  $\{0, 1\}$  it must go through either the left or right hole).

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- Thus we get a loose upper bound of  $3^{N-1} \cdot (N - 2)!$ . With  $N = 9$ , this is  $\approx 33\,000\,000$ .

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- Thus we get a loose upper bound of  $3^{N-1} \cdot (N - 2)!$ . With  $N = 9$ , this is  $\approx 33\,000\,000$ .
- Tighter upper bounds can show the number of valid patterns is more like 30 000, but the loose bound should suffice to show a brute force solution is fast enough.

## Final Solution

Brute force all valid patterns, then count how many patterns result in shoelace length between  $f_{min}$  and  $f_{max}$ .

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Recomputing patterns for each shoelace length rather than storing them. In a slow language this can TLE on big cases.

Statistics: 41 submissions, 6 accepted.

## Problem

Given a tree, find the value of a maximum value path that visits each edge at most  $k$  times, where the value of a path is the weight of every distinct edge in the path.

# F - Agamemnon's Odyssey

## Problem

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## Solution

- The first observation is to see that since the graph is a tree, if  $k \geq 2$ , we can take all edges, since it is always possible to visit every edge of a tree while visiting no edge more than twice; see for example:  
[https://en.wikipedia.org/wiki/Maze\\_solving\\_algorithm#Wall\\_follower](https://en.wikipedia.org/wiki/Maze_solving_algorithm#Wall_follower).

## Problem

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- Otherwise, if  $k = 1$ , we are looking for the longest path in a weighted tree, also known as the *diameter* of the tree.

## Solution

This is a classic problem; solutions can be found online. Two are:

- 1 Run a Dijkstra (DFS) from a node  $u$  to find the farthest away node  $v$ . Run dijkstra again from  $v$  to find the farthest away node  $t$ . The path from  $v$  to  $t$  is the longest in the tree (not too hard to prove yourself!).

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Statistics: 22 submissions, 6 accepted.

# C - Redundant Binary Notation

## Problem

Given a number  $N \leq 10^{16}$  and  $t \leq 100$ , determine the number of representations  $N$  has in a form of binary where each digit can be  $0, 1, \dots$ , or  $t$ .



## Solution

Consider the possible length-three prefixes to valid representations of  $N$  with  $t = 2$ . Here they are listed from smallest to largest, with equal representations on the same line.

```
111
110  022  102
101  021
100  012  020
011
010  002
001
000
```

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```

How many can be valid prefixes? The binary notation takes exactly one. Because we can represent larger values with less digits using digit 2, we could also take the line below it. But we can't take two lines below it! If you fill the remainder of the digits with 2's, the value represented will be less than the binary representation prefix with 0's for the remaining digits.

## Solution

- Example: say the first three digits of  $N$  in binary starts as 010. Then we can also potentially represent  $N$  starting with 001, since  $00122 \dots 2 = 002022 \dots 2 = 010111 \dots 0$ , which is only one less than the maximum number representable starting with 010 in binary.

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- However, if we start 000, then even if all remaining digits are 2, the maximum number we can represent is  $00022 \dots 2 = 001022 \dots 2 = 00111 \dots 0$ , which is less than any binary number starting with 010.

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- However, if we start 000, then even if all remaining digits are 2, the maximum number we can represent is  $00022 \dots 2 = 001022 \dots 2 = 00111 \dots 0$ , which is less than any binary number starting with 010.
- This is true if we list all the prefixes of any length! Furthermore, generalizing to general  $t$  instead of 2, we can show that less than  $t$  lines below the binary representation will allow a valid representation.

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- Let

$Ways(i, d) :=$  number of ways to represent  $i$  in  $d$  digits.

We can try all digits  $0, \dots, t$  in the leading position  $d$  and then recurse on the remaining number. We prune the DP by returning 0 if  $i > t(2^d - 1)$  or  $i < 0$ .

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- The previous analysis shows there will be at most  $O(t \log N)$  explored states of the DP when called with  $\text{Ways}(N, \lfloor \log_2(N) \rfloor + 1)$ .



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## B - Ride-Hailing

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- A trip  $i$  can be matched to trip  $j$  if it is possible for a driver to complete trip  $i$ , then arrive at the start of trip  $j$  at or before the start of trip  $j$ .
- A maximum matching determines the number of drivers necessary. A driver can start at any unmatched trip, then follow the chain of matchings for next trips. The answer is thus  $k$  minus the maximum matching.



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Statistics: 20 submissions, 3 accepted.

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- Let  $DP(\text{board}, \text{turn}, \text{hint}, \text{guesses})$  := probability player whose turn it is wins, given the live cards, the hint word, and how many guesses remain.
- Board is an  $N$ -bit bitmask where a 1 represents a live card.
- The recurrence is as follows: if this is a guessing state ( $\text{guesses} > 0$ ), then we make a uniformly-random choice among live cards associated with  $\text{hint}$ , then recurse on the remaining state.

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  - $O(N)$  on a guessing state. ( $2^N \cdot 2 \cdot M \cdot N$  guessing states.)
  - $O(MN)$  on a clue-giving state. ( $2^N \cdot 2$  clue-giving states.)

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- Number of states:  $2^N \cdot 2 \cdot M \cdot N$ . Time to compute each state:
  - $O(N)$  on a guessing state. ( $2^N \cdot 2 \cdot M \cdot N$  guessing states.)
  - $O(MN)$  on a clue-giving state. ( $2^N \cdot 2$  clue-giving states.)
- Final complexity:  $O(2^N \cdot M \cdot N^2) \approx 700\,000\,000$  iterations, which is a lot, but bit operations are fast and a 15 second time limit generous.

## Solution

- Otherwise, it is a clue-giving state, and we consider all possible remaining hint words and all possible values for  $K$ , taking the maximum.
- Number of states:  $2^N \cdot 2 \cdot M \cdot N$ . Time to compute each state:
  - $O(N)$  on a guessing state. ( $2^N \cdot 2 \cdot M \cdot N$  guessing states.)
  - $O(MN)$  on a clue-giving state. ( $2^N \cdot 2$  clue-giving states.)
- Final complexity:  $O(2^N \cdot M \cdot N^2) \approx 700\,000\,000$  iterations, which is a lot, but bit operations are fast and a 15 second time limit generous.

Statistics: 45 submissions, 1 accepted.

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Statistics: 0 submissions, 0 accepted.

Questions? Comments? Concerns? Email Bryce Sandlund:  
bcsandlund@gmail.com.